WAVE SET UP AND SET DOWN DUE TO A NARROW FREQUENCY WAVE SPECTRUM

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DUE TO
A NARROW FREQUENCY WAVE SPECTRUM

by

Darwin James McReynolds

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A narrow band wave spectrum was applied to theoretical relationships previously developed for set down and set up in an attempt to find second order non-steady solutions for these concepts. The initial effort was to apply this spectrum to the radiation stress tensor using linear wave theory. Another development was attempted by incorporating the spectrum and the solution of the long wave equation into

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the Bernoulli and vertical momentum equations. Results obtained indicate that the solution for mean water level outside the surf zone is composed of a steady component and a periodic unsteady component; the periodic component being of the form of a long wave with a frequency lower than the components of the wave spectrum. The solution for set up is then composed of the same type components. The exact relationships depend on the patching process that is made for the solutions through the breaker line.

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Wave Set Up and Set Down Due to a Narrow Frequency Wave Spectrum

by

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A narrow band wave spectrum was applied to theoretical relationships previously developed for set down and set up in an attempt to find second order non-steady solutions for these concepts. The initial effort was to apply this spectrum to the radiation stress tensor using linear wave theory. Another development was attempted by incorporating the spectrum and the solution of the long wave equation into the Bernoulli and vertical momentum equations. Results obtained indicate that the solution for mean water level outside the surf zone is composed of a steady component and a periodic unsteady component; the periodic component being of the form of a long wave with a frequency lower than the components of the wave spectrum. The solution for set up is then composed of the same type components. The exact relationships depend on the patching process that is made for the solutions through the breaker line.

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TABLE OF SYMBOLS AND ABBREVIATIONS

a	local wave amplitude
c	wave celerity
c_g	wave group velocity
E	total energy density
g	acceleration due to gravity
h	still water depth
h_b	still water depth at breaking
H	wave height
H_b	wave breaking height
k	local wave number
m	slope of beach
M	mass transport of unsteady flow
$\bar{M}$	mass transport of mean flow
M	total mass transport
p	pressure
R_i	friction term
S_{ij}	radiation stress tensor
T_i	horizontal force due to slope of free surface
U	mean velocity component
U	total transport velocity
u'	deviation from mean velocity
u_x	x directed velocity
u_y	y directed velocity
w	vertical velocity

α	angle between wave crest and beach
ε	phase angle
η	wave profile
$\bar{\eta}$	mean water level
η'	deviation from mean water level
$\bar{\eta}_b$	mean water level at breaking
ρ	water density
σ	local radial frequency
ϕ	velocity potential

I. INTRODUCTION

Most of the observable phenomena along a coastline are the direct result of the action of the incoming waves, waves which begin in most instances as a disorganized confused state of the ocean surface, produced by a storm far at sea. In their transit across the vast expanse of the ocean, they begin to sort themselves out and form a somewhat regular oscillation of the ocean surface, the lower frequency oscillations traveling faster and thereby leading the train. This train eventually ends by encountering a beach, where its energy is expended in the form of breaking waves.

Since it is this aspect of the wave's life cycle that influences man the most, considerable effort has been expended investigating this area. Among other effects, it has been observed that waves, in the process of shoaling and eventual breaking, produce a variation in the mean sea level. This variation in sea level has been considered as the primary cause for such nearshore currents as rip currents. The sea surface variation consists of: (a) a gradual depression of the mean sea level beginning offshore and reaching a maximum at the breaker line and (b) inside the surf zone a slope of mean sea level which increases and extends shoreward to a point on the beach higher than the still water line. The depression is termed set down and the slope is called set up.

It is reasonable to expect that variations in the amount of set down or set up along a beach can provide the head to produce a current.

Previous investigations have considered steady state solutions. Experimental results obtained by both Bowen [1967] and Van Dorn [1976] agree quite favorably with the steady state solutions produced by Longuet-Higgins and Stewart [1962] using linear wave theory.

This investigation considers an application of a simple wave spectrum to the existing theories in an attempt to obtain a non-steady solution for the set down and set up phenomena. An opening chapter on background is provided to ensure the necessary understanding of the existing theories and their development. Included is a section on the development of the "radiation stress tensor", a concept which was proved useful by Longuet-Higgins and Stewart [1962] in treating the shoaling process of waves.

Chapter III deals with the application of a simple spectrum to the derivation of set down and set up. First order linear theory and first order theory including a sloping bottom are used to describe the spectral wave components. The final chapters conclude with a comparison of the steady state solutions produced by the earlier work and the unsteady results obtained here. The numerical results are compared with the results for the steady case given by Bowen [1967].

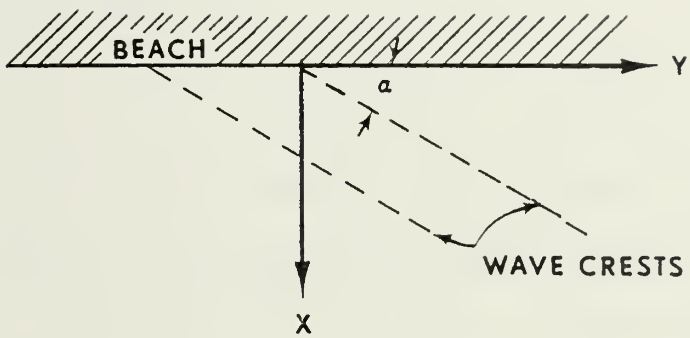
II. BACKGROUND

Changes in mean sea level near a shoreline have been studied both theoretically and experimentally. A theoretical framework was formulated by Longuet-Higgins and Stewart [1962, 1963, 1964] which dealt with the excess momentum flux due to the presence of unsteady wave motion and which they termed "radiation stress". Longuet-Higgins and Stewart were able to define many of the shoaling effects of a train of waves including wave set down and set up utilizing this radiation stress concept. This chapter reviews the development of the radiation stress tensor and its relationship to the concepts of set down and set up. It also includes a direct approach used by Longuet-Higgins [1967] to derive an expression for set down utilizing the vertical momentum equation and the Bernoulli equation.

A. DEVELOPMENT OF THE RADIATION STRESS TENSOR

Since the approach of Longuet-Higgins and Stewart is rather lengthy and tends to obscure the concepts involved, the later and more systematic approach by Phillips [1966] is used. The development of the radiation stress tensor and the resulting phenomena of set down and set up is kept as general as possible. The coordinate system is given by Figure (1) where the x-axis is perpendicular to the shore, the y-axis is parallel to the shore, and the z-axis is

PLAN VIEW



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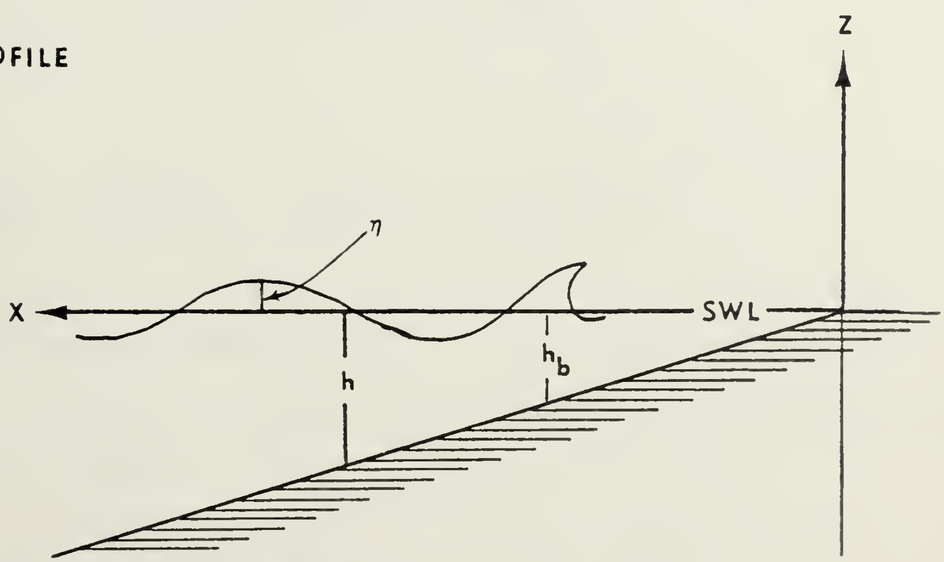

FIGURE 1: Coordinate System

vertically upward from the still water level. The governing equations are the continuity equation and the horizontal momentum equations.

The continuity equation is given by

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u_i}{\partial x_i} + \frac{\partial \rho w}{\partial z} = 0 , \qquad i = 1, 2 \tag{1}$$

where 1, 2 refer to the x, y components. The horizontal velocity u_i is composed of a mean flow component, U_i, and a fluctuating component representing the deviation from mean flow, u_i', such that $u_i = U_i + u_i'$. Since there is no mean flow in the vertical, $w = w'$. Multiplying the continuity equation by u_i and adding this result to the horizontal momentum equation

$$\rho \frac{\partial u_i}{\partial t} + \rho u_j \frac{\partial u_i}{\partial x_j} + \rho w \frac{\partial u_i}{\partial z} = - \frac{\partial p}{\partial x_i} + R_i \tag{2}$$

produces

$$\rho \frac{\partial u_i}{\partial t} + \frac{\partial \rho\ u_i u_j}{\partial x_j} + \frac{\partial \rho\ u_i w}{\partial z} = - \frac{\partial p}{\partial x_i} + R_i \tag{3}$$

Integrating over depth from $-h$ to η, using Leibnitz's rule and applying the kinematic free surface and bottom boundary condition yields

$$\frac{\partial}{\partial t} \int_{-h}^{\eta} \rho u_i dz + \frac{\partial}{\partial x_i} \int_{-h}^{\eta} \rho u_i u_j dz + \frac{\partial}{\partial x_i} \int_{-h}^{\eta} p dz$$
$$+ \ p_{-h} \frac{\partial(-h)}{\partial x_i} = R_i \ .$$

By time averaging this equation term by term and by making the following definitions

$$\bar{M}_i = \overline{\int_{-h}^{\eta} \rho U_i dz} = \rho U_i(\bar{\eta} + h) ,$$
$$M_i = \overline{\int_{-h}^{\eta} \rho u_i' dz} , \tag{4}$$
$$\tilde{M}_i = \bar{M}_i + M_i ,$$
$$\tilde{U}_i = \frac{\tilde{M}_i}{\rho(\bar{\eta} + h)} = U_i + \frac{M_i}{\rho(\bar{\eta} + h)} ,$$

the following expression is obtained:

$$\frac{\partial \tilde{M}_i}{\partial t} + \frac{\partial}{\partial x_j} \{\tilde{U}_i \tilde{M}_i + \overline{\int_{-h}^{\eta} [\rho u_i' u_j' + p\delta_{ij}] dz} - \frac{M_i M_j}{\rho(\bar{\eta} + h)}$$
$$- \frac{1}{2} \rho g(\bar{\eta} + h)\delta_{ij}\}$$
$$= T_i + R_i . \qquad \delta_{ij} = 0 \text{ for } i \neq j \qquad (5)$$
$$= 1 \text{ for } i = j$$

The first term on the left hand side is the local change in horizontal momentum flux. In the braces, the first term represents the momentum flux produced by the steady state flow. The last three terms in the braces contain all the unsteady contribution to the momentum flux with the hydrostatic effect subtracted out; this is the momentum flux due

to the unsteady motion or the excess momentum flux referred to as the radiation stress tensor:

$$S_{ij} = \overline{\int_{-h}^{\eta} [\rho u_i' u_j' + p\delta_{ij}] dz} - \frac{M_i M_j}{\rho(\bar{\eta}+h)} - \frac{1}{2}\rho g(\bar{\eta}+h)^2 \delta_{ij} \quad . \tag{6}$$

Equation (5) is then simplified to

$$\frac{\partial}{\partial t}\tilde{M}_i + \frac{\partial}{\partial x_j}\{\tilde{U}_i\tilde{M}_i + S_{ij}\} = T_i + R_i \quad . \tag{7}$$

T_i represents the horizontal force produced by the slope of the free surface and is given by

$$T_i = -\rho g(\bar{\eta}+h)\frac{\partial\bar{\eta}}{\partial x_i} \quad . \tag{8}$$

Outside the surf zone it is assumed $\bar{\eta} << -h$ and (8) reduces to

$$T_i = -\rho g h \frac{\partial\bar{\eta}}{\partial x_i} \quad . \tag{9}$$

R_i is the averaged and integrated frictional stress term. These equations are general and apply to all kinds of steady and unsteady motion. The only simplifying assumption is that mean flow is uniform over depth. The advantage of using the "radiation stress" technique to solve physical problems is that the second order effects are obtained using first order theory.

B. WAVE SET DOWN

To present the concept of set down, consider only the x-component of horizontal momentum flux equation (7) with

waves that propagate shoreward from deep water with their crests making an arbitrary angle α with the shoreline. For simplicity, the bottom is assumed to be composed of parallel contours so that gradients in the y-direction are zero. The bottom slope is allowed to vary only gradually so that energy reflection from the shore may be neglected and the shoaling effects caused by changes in the bottom can be considered in a step-like fashion. By neglecting the frictional effects and assuming that any current gradient in the x-direction is small, (7) can be written for outside the surf zone as

$$\frac{\partial \tilde{M}_x}{\partial t} + \rho g h \frac{\partial \bar{\eta}}{\partial x} = - \frac{\partial S_{xx}}{\partial x} . \qquad (10)$$

Since (10) is an equation in two unknowns ($\tilde{M}_x$ and $\bar{\eta}$), a second equation is required for a solution. To provide it, the continuity equation, (1), is vertically integrated over depth by the use of Leibnitz's rule. This produces the conservation of mass flux equation as given by Phillips [1966] where gradients in the y-direction are zero.

$$\rho \frac{\partial \bar{\eta}}{\partial t} + \frac{\partial \tilde{M}_x}{\partial x} = 0 . \qquad (11)$$

Utilizing linear wave theory, the radiation stress tensor is proportional to the square of the local wave amplitude, a^2, or to the total energy. Therefore, following the method of Longuet-Higgins and Stewart [1962], the applied force of the

system travels with the speed of the energy, i.e., at the group velocity, c_g. Hence the transformation $\partial/\partial t + c_g\, \partial/\partial x = 0$ can be applied. Equations (10) and (11) become

$$- c_g \frac{\partial \tilde{M}_x}{\partial x} + \rho g h \frac{\partial \bar{\eta}}{\partial x} = - \frac{\partial S_{xx}}{\partial x} , \qquad (12)$$

$$\frac{\partial \tilde{M}_x}{\partial x} - c_g \frac{\partial \bar{\eta}}{\partial x} = 0 , \qquad (13)$$

for which the solution is

$$\frac{\partial \bar{\eta}}{\partial x} = \frac{1}{\rho} \frac{1}{(gh - c_g^2)} \frac{\partial S_{xx}}{\partial x} , \qquad (14)$$

or on integration

$$\bar{\eta} = \frac{1}{\rho} \frac{1}{(gh - c_g^2)} S_{xx} . \qquad (15)$$

Since $c_g^2 \rightarrow gh$ in shallow water, equation (15) is a non-steady state solution which implies that the mean water level increases negatively without bound as the wave moves into shallow water. The explanation offered by Longuet-Higgins and Stewart [1962] for this apparent resonant condition is that its effect takes time to build and the energy involved is dissipated prior to it reaching any significance by the breaking of the wave.

By imposing steady state conditions and describing the unsteady motion using linear wave theory, Longuet-Higgins and Stewart [1962] subsequently developed the following situation for $\bar{\eta}$:

$$\bar{\eta} = - \frac{1}{2} \frac{a^2 k}{\sinh 2kh} . \qquad (16)$$

This is a second order equation in terms of the local depth, amplitude, and wave number. It is apparent that as the wave train approaches the point of breaking, $\bar{\eta}$ decreases and there is a set down of the mean water level. The set down increases to the point where the wave breaks and other assumptions regarding $\bar{\eta}$ must be applied.

Longuet-Higgins [1967] also derived this solution in another manner without referring to the radiation stress term. By integrating the vertical momentum equation over depth and time averaging, the total average vertical momentum is obtained:

$$(\bar{p} - \rho\overline{w}^2)_{z=0} - \rho g\bar{\eta} = 0 \; . \qquad (17)$$

A second equation used is Bernoulli's integral

$$p + \frac{1}{2}\,\rho(u_x^{\;2} + u_y^{\;2} + w^2) + \rho g z + \rho\,\frac{\partial\phi}{\partial t} = 0 \; , \qquad (18)$$

with the restriction that the flow be irrotational. By setting z = 0 and time averaging, (18) becomes

$$\bar{p}_{z=0} + \frac{1}{2}\,\rho\,(\bar{u}_x^{\;2} + \bar{u}_y^{\;2} + \overline{w}^2)_{z=0} + C = 0 \; , \qquad (19)$$

where C is at most a constant. From (17) and (19), $\bar{p}_{z=0}$ can be eliminated giving

$$\bar{\eta} = -\frac{1}{2}\,\frac{1}{g}\,(\bar{u}_x^{\;2} + \bar{u}_y^{\;2} - \overline{w}^2)_{z=0} + C = 0 \; . \qquad (20)$$

From this, the difference in mean sea level is obtained at two different points $(x_1, y_1, 0)$ and $(x_2, y_2, 0)$ thereby eliminating the constant C,

$$\Delta\bar{\eta} = -\frac{1}{2g}\left[(\bar{u}_x^{\,2} + \bar{u}_y^{\,2} - \bar{w}^2)_{z=0}\right]_2^1 . \qquad (21)$$

The velocities are expressed applying linear wave theory using the following relationships:

$$u_x = \frac{a\sigma \cos\alpha}{\sinh kh} \cosh k(z-h)\cos(kx'-\sigma t+\varepsilon) ,$$

$$u_y = \frac{a\sigma \sin\alpha}{\sinh kh} \cosh k(z-h)\cos(kx'-\sigma t+\varepsilon) , \qquad (22)$$

$$w = \frac{a\sigma}{\sinh kh} \sinh k(z-h)\sin(kx'-\sigma t+\varepsilon) ,$$

where $x' = x\cos\alpha + y\sin\alpha$ and ε denotes an arbitrary constant phase angle. Substituting equations (22) into the right hand side of (21) yields

$$\frac{1}{2g}(\bar{u}_x^{\,2} + \bar{u}_y^{\,2} - \bar{w}^2) = \frac{\alpha^2\sigma^2}{4g\sinh^2 kh} , \qquad (23)$$

and the difference in mean sea level is then obtained as

$$\Delta\bar{\eta} = -\frac{a^2 k}{2\sinh 2kh}\Big]_2^1 . \qquad (24)$$

By assuming the point $(x_2, y_2, 0)$ is in infinitely deep water so that $\bar{\eta}_2 = 0$, the value for $\bar{\eta}$ at the point $(x_1, y_1, 0)$ becomes

$$\bar{\eta} = -\frac{1}{2}\frac{a^2 k}{\sinh 2kh} . \qquad (25)$$

This solution is very simple and straightforward, with the same assumptions as before applying to the bottom slope and local depth h. This solution is plotted in Figure (2) with the strong correlation to laboratory results given by Bowen, Innman, and Simmons [1968].

As waves progress shoreward they travel from deep water to shallow water. The transition in the linear equation can be made by utilizing the conservation of energy flux ($E \cdot c_g$ = constant). However, when a wave approaches the point of breaking,the wave steepens beyond that allowed by linear theory and another approximation must be made. It is assumed that at breaking

$$H_b = \gamma h_b , \qquad (26)$$

where γ is a proportionality constant which is generally assumed to be equal to 0.78 borrowed from solitary wave theory. Then the mean sea level depression at the breaking point, $\bar{\eta}_b$, is given by

$$\bar{\eta}_b = -\frac{\gamma}{16} H_b . \qquad (27)$$

C. SET UP

Set up is a phenomenon that occurs in the surf zone shoreward of the breaker line. Because energy decreases shoreward in this zone, a different formulation for the wave amplitude must be used than the local value used before. It is assumed that the breakers being considered are of the spilling type which retain their harmonic characteristics and gradually decrease in amplitude as they progress shoreward such that wave height is defined by

$$H = \gamma(\bar{\eta} + h) \ , \tag{28}$$

where γ is the proportionality constant introduced earlier. Again by neglecting frictional effects, considering only x-direction motion, and by assuming that any current gradients in this direction are small, equation (7) yields

$$\frac{\partial \tilde{M}_x}{\partial t} + \frac{\partial S_{xx}}{\partial x} = -\rho g(\bar{\eta} + h)\frac{\partial \bar{\eta}}{\partial x} \ . \tag{29}$$

where it cannot be assumed that $\bar{\eta} << h$. In this equation, S_{xx} can be determined by using linear wave theory and the shallow water approximation giving

$$S_{xx} = \frac{3}{2} E \ ,$$

which inside the surf zone is equivalent to

$$S_{xx} = \frac{3}{16} \rho g \gamma^2 (h + \bar{\eta})^2 \ .$$

By assuming steady state conditions, (29) can be written as

$$\frac{\partial}{\partial x} \frac{3}{2} E = - \rho g(\bar{\eta} + h) \frac{\partial \bar{\eta}}{\partial x} .$$

Since energy decreases shoreward, it is evident that $\bar{\eta}$ increases and produces a set up effect. Solving for $\bar{\eta}$

$$\bar{\eta} = - kh + C ,$$

where

$$k = \frac{1}{1 + \frac{1}{\frac{3}{8} \gamma^2}} .$$

By applying the value of $\bar{\eta}_b$ at the breaker line, the constant C can be eliminated and

$$\bar{\eta} = k(h_b - h) + \bar{\eta}_b . \qquad (30)$$

Where a beach has a constant slope given by h = mx, the set up also has a constant slope proportional to the beach slope. It should be noted that the simple solution obtained by application of Bernoulli's integral outside the surf zone is not applicable since the motion is no longer irrotational inside the surf zone.

III. APPLICATION OF A NARROW BAND WAVE SPECTRUM TO THE EXISTING THEORIES

The expressions just considered were made as general as possible in order to afford a simple physical understanding. Only a monochromatic wave train was considered and only steady state solutions were allowed. In this chapter, unsteady terms are introduced into these expressions by applying linear wave theory in the form of a simple two frequency component wave spectrum simulating a narrow band wave spectrum. In this manner an unsteady second order expression for the mean water level, $\bar{\eta}$, is derived. Next, using this simple spectrum, another solution for $\bar{\eta}$ is derived using Longuet-Higgins' application of the Bernoulli equation. Then by incorporating this spectrum into Iwagaki's first order solution for a sloping bottom [1972] and applying the Longuet-Higgins' approach, a third solution for $\bar{\eta}$ is obtained. Finally, the application of the spectrum to the set up phenomenon inside the surf zone is considered.

A. WAVE SET DOWN

Consider a simplified wave spectrum, specifically consisting of two waves which have nearly the same frequency and wave number such that $\sigma_1 - \sigma_2 = \Delta\sigma$, and $k_1 - k_2 = \Delta k$ are very small. Also for simplicity allow their respective amplitudes to be identical and equal to $\frac{a}{\sqrt{2}}$. This amplitude is chosen in order that the variance of the simple wave spectrum and

the variance of a monochromatic wave amplitude are the same. The wave components are to be added linearly sucn that the particle velocities become $u = u_1 + u_2$. If the first order velocity potential is given locally by

$$\phi = -\frac{ag}{\sqrt{2}\,ck}\,\frac{\cosh k(h+z)}{\cosh kh}\,\sin(k_i x_i - \sigma t)\ , \quad i = 1,\ 2 \tag{31}$$

where k_i is used to indicate direction, i.e. $k_x/k = \cos\alpha$ and $k_y/k = \sin\alpha$, then in general $u_i = -\frac{\partial\phi}{\partial x_i}$ or

$$u_i = \frac{ag}{\sqrt{2}\,c}\,\frac{k_i}{k}\,\frac{\cosh k(h+z)}{\cosh kh}\,\cos(k_i x_i - \sigma t)\ . \tag{32}$$

Since $u_i = u_{i_1} + u_{i_2}$ in this case, u_i becomes

$$u_i = \frac{ag}{\sqrt{2}\,c}\Big[\frac{k_{i_1}}{k_1}\,\frac{\cosh k_1(h+z)}{\cosh k_1 h}\,\cos(k_{i_1} x_i - \sigma_1 t)$$

$$+ \frac{k_{i_2}}{k_2}\,\frac{\cosh k_2(h+z)}{\cosh k_2 h}\,\cos(k_{i_2} x_i - \sigma_2 t)\ . \tag{33}$$

Since $k_1 - k_2 = \Delta k$ and $\sigma_1 - \sigma_2 = \Delta\sigma$, the subscripts may be dropped and (28) can be written as

$$u_i = \frac{ag}{\sqrt{2}\,c}\Big\{\frac{k_i}{k}\,\frac{\cosh k(h+z)}{\cosh kh}\,\cos(k_i x_i - \sigma t)$$

$$+\Big(\frac{k_i+\Delta k_i}{k+\Delta k}\Big)\,\frac{\cosh[(k+\Delta k)(h+z)]}{\cosh(k+\Delta k)h}\cos[(k_i+\Delta k_i)x_i-(\sigma+\Delta\sigma)t]\Big\}\ .$$

(34)

This simplifies to

$$u_i = \frac{2ag}{\sqrt{2}\,c}\,\frac{k_i}{k}\,\frac{\cosh k(h+z)}{\cosh kh}\cos\left(\frac{\Delta k_i}{2}x_i - \frac{\Delta\sigma}{2}t\right)\cos(k_i x_i - \sigma t) \quad . \tag{35}$$

Similarly, the particle velocity in the z-direction, w, is obtained and is

$$w = \frac{2ag}{\sqrt{2}\,c}\,\frac{\sinh k(h+z)}{\cosh kh}\cos\left(\frac{\Delta k_i}{2}x_i - \frac{\Delta\sigma}{2}t\right)\sin(k_i x_i - \sigma t) \quad . \tag{36}$$

These expressions state that the particle velocities are sinusoidal with a periodically varying amplitude. The modulating wave is of low frequency and with a wavelength that is much longer than those of the original two base waves.

The unsteady velocities specified by (35) and (36) are substituted into (6) and the radiation stress tensor determined. The time averaging process is carried out over the shorter periods of the base waves. Since the modulating wave is of such low frequency, its effects remain in the solution. Specifically, equation (6) becomes term by term

$$\overline{\int_{-h}^{\eta} \rho u_i' u_j' dz} = E\,\frac{c_g}{c}\,\frac{k_i k_j}{k^2}\cos\left(\frac{\Delta k_i x_i}{2} - \frac{\Delta\sigma}{2}t\right)\cos\left(\frac{\Delta k_j x_j}{2} - \frac{\Delta\sigma}{2}t\right) \ , \tag{37}$$

where $E = \frac{1}{2}\rho g a^2$ is the total energy density, and c_g and c are the group velocity and phase velocity respectively;

$$\int_{-h}^{\eta} p\,dz = \rho g(\bar{\eta}+h)^2 + \frac{E}{2} - E\left(\frac{c_g}{c} - 2\right)\cos^2\left(\frac{\Delta k_i}{2}x_i - \frac{\Delta\sigma}{2}t\right) \ ; \tag{38}$$

and the remaining unspecified term becomes

$$\frac{M_i M_j}{\rho(\bar{\eta}+h)} = \frac{1}{\rho(\bar{\eta}+h)} \frac{E^2}{c^2} \frac{k_i k_j}{k^2} \cos(\frac{\Delta k_i}{2} x_i - \frac{\Delta\sigma}{2} t)\cos(\frac{\Delta k_j}{2} w_j - \frac{\Delta\sigma}{2} t) \quad . \tag{39}$$

Substituting (37), (38), and (39) into (6), the radiation stress tensor becomes:

$$S_{ij} = E \frac{c_g}{c} \frac{k_i k_j}{k^2} \cos(\frac{\Delta k_i}{2} x_i - \frac{\Delta\sigma}{2} t)\cos(\frac{\Delta k_j}{2} x_j - \frac{\Delta\sigma}{2} t) + \frac{E}{2}$$

$$- E(\frac{c_g}{c} - 2) \cos^2(\frac{\Delta k_i}{2} x_i - \frac{\Delta\sigma}{2} t) \tag{40}$$

$$- \frac{1}{\rho(\bar{\eta}+h)} \frac{E^2}{c^2} \frac{k_i k_j}{k^2} \cos(\frac{\Delta k_i}{2} x_i - \frac{\Delta\sigma}{2} t)\cos(\frac{\Delta k_j}{2} x_j - \frac{\Delta\sigma}{2} t) \quad .$$

Since only second order effects are being considered and $E^2 = 0(a^4)$ the last term may be neglected. It is obvious that the proper choice of assumptions will simplify (40) immensely, e.g. consider only the x-direction components in shallow water,

$$S_{xx} = [E(\frac{k_x}{k})^2 + E] \cos^2(\frac{\Delta k_i}{2} x_i - \frac{\Delta\sigma}{2} t) + \frac{E}{2} \; ,$$

and if the waves approach the shoreline such that the angle $\alpha = 0$, this expression further reduces to

$$S_{xx} = E[1 + \cos(\Delta k_i x_i - \Delta\sigma t) + \frac{E}{2} \; . \tag{41}$$

This last expression states that the radiation stress, in this circumstance, is composed of a steady stress term and of one that is periodic.

This derived form of the radiation stress is now used to find a solution for $\bar{\eta}$. Recalling equations (10) and (11), and as before neglecting the friction term and assuming that the current gradients in the x-direction are small, the x-component of these expressions become:

$$\frac{\partial \tilde{M}_x}{\partial t} + \frac{\partial S_{xx}}{\partial x} = -\rho g h \frac{\partial \bar{\eta}}{\partial x}$$

and
$$\frac{\partial \tilde{M}_x}{\partial x} + \rho \frac{\partial \bar{\eta}}{\partial t} = 0 \ .$$

By cross differentiating, the $\tilde{M}_x$ term can be eliminated and

$$-\rho \frac{\partial^2 \bar{\eta}}{\partial t^2} + \rho g \frac{\partial}{\partial x}\left[h \frac{\partial \bar{\eta}}{\partial x}\right] = -\frac{\partial^2 S_{xx}}{\partial x^2} \tag{42}$$

is obtained. Assuming a flat bottom so that h is considered constant, (42) becomes

$$-\rho \frac{\partial^2 \bar{\eta}}{\partial t^2} + \rho g h \frac{\partial^2 \bar{\eta}}{\partial x^2} = -\frac{\partial^2 S_{xx}}{\partial x^2} \ . \tag{43}$$

This is obviously the long wave equation which is being forced by the radiation stress term. It has a solution in the homogeneous case of

$$\bar{\eta}_c = a \cos(\Delta k_x x - \Delta\sigma t) \ .$$

For a particular solution, make the assumption that led to equation (41), i.e., consider shallow water and $\alpha = 0$. Then assume a particular solution of the form

$$\eta_p = A \cos(\Delta k_x x - \Delta\sigma t) + B \sin(\Delta k_x x - \Delta\sigma t) \ .$$

By substituting this into equation (43), the solution for A and B are obtained

$$A = - \frac{E\Delta k_x^{\ 2}}{\rho(\Delta\sigma^2 - gh\Delta k^2)} \ ,$$

$$B = 0 \ .$$

The solution for $\bar{\eta}$ is then

$$\bar{\eta} = [a - \frac{\frac{1}{2} a^2 g}{(\frac{\Delta\sigma^2}{\Delta k_x^2} - gh)}] \cos(\Delta k_x x - \Delta\sigma t) \ , \qquad (44)$$

which is a wave with an amplitude that is proportional to the steady state solution of Longuet-Higgins and Stewart, equation (15).

If the same assumptions as before are made except that the bottom is allowed to vary as $h = mx$ where m is some constant slope, a more difficult problem is encountered. Equation (42) becomes

$$-\rho \frac{\partial^2 \bar{\eta}}{\partial t^2} + \rho g m \frac{\partial \bar{\eta}}{\partial x} + \rho g m x \frac{\partial^2 \bar{\eta}}{\partial x^2} = - \frac{\partial^2 S_{xx}}{\partial x^2} \ . \qquad (45)$$

This equation again has the characteristics of the long wave equation that is being forced by the radiation stress. An

analytical solution of this equation would provide a general form for $\bar{\eta}$; however, since the depth is allowed to vary with x, the expression for the radiation stress term becomes complicated and obtaining a particular solution for (45) becomes difficult.

B. WAVE SET DOWN USING THE BERNOULLI INTEGRAL

Recalling that the second method of Longuet-Higgins to determine a solution for $\bar{\eta}$ resulted in the equation

$$\Delta\bar{\eta} = \frac{1}{2g} \left[(\bar{u}_x^{\,2} + \bar{u}_y^{\,2} - \bar{w}^2)_{z=0} \right]_2^1 ,$$

a second unsteady solution may be obtained by applying the simple wave spectrum. From equations (35) and (36) $\bar{u}_x^{\,2}$, $\bar{u}_y^{\,2}$, and $\bar{w}^2$ become

$$\bar{u}_x^{\,2} = \frac{a^2g^2}{c^2} \left(\frac{k_x}{k}\right)^2 \frac{\cosh^2 k(h+z)}{\cosh^2 kh} \cos^2\left(\frac{\Delta k_x}{2} x - \frac{\Delta\sigma}{2} t\right) ,$$

$$\bar{u}_y^{\,2} = \frac{a^2g^2}{c^2} \left(\frac{k_y}{k}\right)^2 \frac{\cosh^2 k(h+z)}{\cosh^2 kh} \cos^2\left(\frac{\Delta k_y}{2} y - \frac{\Delta\sigma}{2} t\right) ,$$

$$\bar{w}^2 = \frac{a^2g^2}{c^2} \frac{\sinh^2 k(h+z)}{\cosh^2 kh} \cos^2\left(\frac{\Delta k_x}{2} x + \frac{\Delta k_y}{2} y - \frac{\Delta\sigma}{2} t\right) ,$$

and

$$-\frac{1}{2g}(\bar{u}_x^{\,2}+\bar{u}_y^{\,2}-\bar{w}^2)_{z=0} = -\frac{a^2g^2}{2c^2}\Big\{\left(\frac{k_x}{k}\right)^2\cos^2\left(\frac{\Delta k_x}{2} x - \frac{\Delta\sigma}{2} t\right) + \left(\frac{k_y}{k}\right)^2\cos^2\left(\frac{\Delta k_y}{2} y - \frac{\Delta\sigma}{2} t\right)$$

$$- \tanh^2 kh \cos^2\left(\frac{\Delta k_x}{2} x + \frac{\Delta k_y}{2} y - \frac{\Delta\sigma}{2} t\right)\Big\} . \quad (46)$$

Again assuming $\alpha = 0$ so that $k_x/k = 1$, and $k_y/k = 0$ and recalling that $c^2 = g/k \tanh kh$, (46) becomes

$$\bar{\eta} = -\frac{1}{2}\frac{a^2 k}{\sinh 2kh}\left[1 + \cos(\Delta k_x x - \Delta\sigma t)\right] . \quad (47)$$

This solution is identical to the solution of Longuet-Higgins (25) except for the unsteady term. The set down fluctuates at $\Delta\sigma = \sigma_1 - \sigma_2$; which is the difference of the radial frequencies of the two component wave spectrum. This low frequency oscillation is often described as surf beat.

The amplitude of $\bar{\eta}$ varies from twice the amplitude of (25) when the two base waves are in phase, to zero when the base waves are out of phase. Hence the set down can be considered the superposition of a steady component and an unsteady component.

C. WAVE SET DOWN ON A SLOPING BEACH

In an effort to more accurately represent shoaling wave transformations, Iwagaki [1972] implicitly considers constant bottom slope resulting in solutions to the long wave equation involving the Bessel functions. He obtained first order solutions for η and u in terms of the asymptotic form of the Bessel and Neumann functions. It seems reasonable to assume that by incorporating these solutions into the Bernoulli integral, a second order solution for $\bar{\eta}$ can be obtained which includes a more realistic representation of the bottom effects. Assuming shallow water conditions with the depth given by $h = mx$ and progressive waves,

$$u_x = \frac{a}{\sqrt{2}}\sqrt{\frac{g}{mx}}\left[J_1(\chi)\sin\sigma t + N_1(\chi)\cos\sigma t\right] \quad (48)$$

and

$$\eta = \frac{a}{\sqrt{2}} [J_o(\chi)\cos \sigma t + N_o(\chi)\sin \sigma t] , \tag{49}$$

where $\chi^2 = 4 \frac{\sigma^2 x}{gm}$. The asymptotic expansions of the Bessel and Neumann functions are:

$$\begin{aligned} J_n(\omega) &\sim \sqrt{\frac{2}{\pi\omega}} \cos(\omega - \frac{n\pi}{2} - \frac{\pi}{4}) , \\ N_n(\omega) &\sim \sqrt{\frac{2}{\pi\omega}} \sin(\omega - \frac{n\pi}{2} - \frac{\pi}{4}) . \end{aligned} \tag{50}$$

Substituting these expressions into (48) and simplifying yields

$$u_x = \frac{a}{\sqrt{2}} \sqrt{\frac{2}{\pi\chi}} [\sin(\chi - \frac{3\pi}{4} - \sigma t)] , \tag{51}$$

and from the kinematic free surface boundary condition

$$w_{\bar{\eta}} = \frac{\partial\bar{\eta}}{\partial t} = \frac{a}{\sqrt{2}}\sigma \sqrt{\frac{\sqrt{gmx}}{\pi\sigma x}} [\sin(\chi - \frac{\pi}{4} - \sigma t)] . \tag{52}$$

Introducing the wave spectrum as before, squaring and time averaging the result, produces

$$\bar{u}_x{}^2{}_{z=0} = \frac{a^2 g\sqrt{gmx}}{2mx^2\pi\sigma} [1 + \cos(\frac{2\Delta\sigma x}{\sqrt{gmx}} + \Delta\sigma t)] , \tag{53}$$

and

$$\bar{w}^2{}_{z=\bar{\eta}} = \frac{a^2\sigma^2\sqrt{gmx}}{2x\pi\sigma} [1 + \cos(\frac{2\Delta\sigma x}{\sqrt{gmx}} + \Delta\sigma t)] . \tag{54}$$

By assuming that $\bar{\eta}_2$ is in infinitely deep water as before, the second order solution for $\bar{\eta}$ is

$$\bar{\eta} = - \frac{a^2\sqrt{gmx}}{2g\pi\sigma x} \left[\frac{g}{mx} - \sigma^2\right]\left[1+\cos\left(\frac{2\Delta\sigma x}{gmx} + \Delta\sigma t\right)\right] . \quad (55)$$

D. WAVE SET UP

Because the nature of the solution for $\bar{\eta}$ using linear theory was the superposition of a steady state component and an unsteady component, it is reasonable to assume that this condition persists across the breaker line and the set up resulting in the surf zone from the wave spectrum has a similar makeup. The steady state component is then derived separately in the same manner as in Chapter II with the radiation stress being now defined as the steady state portion of equation (41), or

$$S_{xx} = \frac{3}{2} E , \quad (56)$$

and again

$$\bar{\eta}_s = - kh + C , \quad (57)$$

where this time the proportionality constant is again

$$k = \frac{1}{1 + \frac{1}{\frac{3}{8}\gamma^2}} .$$

Since $\bar{\eta}_b = \frac{1}{16} \gamma H_b$, (57) becomes

$$\bar{\eta}_s = k(h_b - h) + \bar{\eta}_b . \quad (58)$$

The unsteady component is assumed to be periodic in character; however, unlike the higher frequency waves of the train, observations suggest that it is not attenuated as it approaches the shoreline but is reflected to some extent. The formulation is then that of the long wave equation as given by Stoker [1966],

$$\frac{\partial^2 \bar{\eta}}{\partial t^2} - gh \frac{\partial^2 h}{\partial x^2} - gh \frac{\partial h}{\partial x} \frac{\partial \bar{\eta}}{\partial x} = 0 , \tag{59}$$

and the solution is given by Guza and Bowen [1977] as

$$\bar{\eta} = a\{J_o(\chi)\sin \Delta\sigma t + N_o(\chi)\cos \Delta\sigma t +$$

$$r[J_o(\chi)\sin(\Delta\sigma t-\varepsilon)-N_o(\chi)\cos(\Delta\sigma t-\varepsilon)]\} \tag{60}$$

where a is the amplitude of the incoming wave (in this case $a = \bar{\eta}_b$), r is the reflection coefficient, and ε a phase shift in the reflected wave. The frequency of this wave must be the same as that of the modulating wave derived outside the surf zone or $\Delta\sigma$. By substituting the asymptotic form of the Bessel and Neumann functions,

$$\bar{\eta}_u = \bar{\eta}_b \sqrt{\frac{2}{\pi\chi}} \{\cos(\chi-\Delta\sigma t - \frac{\pi}{4}) + r \cos(\chi-\Delta\sigma t-\theta)\} , \tag{61}$$

where $\theta = \frac{\pi}{4} + \varepsilon$. Since the set up shoreward was assumed to be a composite of the two solutions (58) and (61),

$$\bar{\eta} = k(h_b-h)+\bar{\eta}_b \{1 + \sqrt{\frac{2}{\pi\chi}}[\cos(\chi-\Delta\sigma t- \frac{\pi}{4})+r\cos(\chi-\Delta\sigma t-\theta)]\} . \tag{62}$$

IV. COMPARISONS

The results obtained here are compared with the laboratory experiments performed by Bowen, et al.[1968]; Figure 2. They obtained experimental results from a controlled wave tank experiment, and in each case only a monochromatic wave was considered. Their results compare quite favorably to the steady state solutions of Longuet-Higgins and Stewart [1962].

Since it was assumed that the variance of the simple wave spectrum was equal to the variance of the monochromatic wave, the derived expressions differ from existing theory by that amount which is contributed by the modulating wave of the unsteady portion. Comparing Longuet-Higgins' solution (25)

$$\bar{\eta} = -\frac{1}{2}\frac{a^2 k}{\sinh 2kh}$$

with the unsteady solution (47)

$$\bar{\eta} = -\frac{1}{2}\frac{a^2 k}{\sinh 2kh}[1 + \cos(\Delta k_x x - \Delta\sigma t)] ,$$

it is found that, by averaging the effect of the unsteady component, they are the identical. As mentioned earlier, the total effect of the unsteady motion is, in this case, to produce a set down that oscillates periodically from twice the steady solution to zero times it, i.e., no set

down at all. Thus a group of waves which are close in frequency and wavelength may be said to produce in the near shore region a fluctuation in the mean sea level seaward of the breaker line. This is also demonstrated by the unsteady solution obtained by allowing for a sloping bottom, equation (55). It too consists of a steady component and an oscillating component.

The initial solution, equation (44), on the other hand is comprised of only an oscillating component. However, this solution was derived in a simplistic manner by assuming that the depth remained constant and by solving the resulting differential equation. While this demonstrates that the mean sea level oscillates in a wave-like fashion when driven by a group of waves, it does little to provide insight into the near shore region.

The set down can be expressed in shallow water in terms of deep water conditions by using conservation of energy flux,

$$E\, c_g = \frac{1}{2}\,\rho g a^2\, c_g = \text{constant}\ ,$$

where

$$\frac{1}{2}\,\rho g a_o^{\,2}\,\frac{g}{2\sigma} = \frac{1}{2}\,\rho g a_s^{\,2}\sqrt{gh}$$

or

$$a_s^{\,2} = \frac{a_o^{\,2}}{2}\,\frac{g}{\sigma}\,\frac{1}{\sqrt{gh}}\ .$$

The subscript o refers to deep water and the subscript s refers to shallow water. The set down in shallow water by equation (25) is then given by

$$\bar{\eta}_1 = \frac{a_o^2 g^{1/2} (mx)^{-3/2}}{4\sigma} , \qquad (63)$$

where in shallow water sinh 2kh → 2kh and h = mx for a constant sloping bottom.

The set down derived from linear theory can be compared to that derived from linear theory for a sloping bottom, equation (55), by also expressing the shallow water amplitude for sloping bottom solutions in terms of deep water conditions. This is accomplished by patching Stokes solution at the off shore to the shallow ater solution. For a smooth match it is required that, from Friedrich [1948],

$$a_s^2 = \frac{a_o^2 \pi}{2m} .$$

The steady state solution of (55) can now be expressed in terms of deep water conditions

$$\begin{aligned} \bar{\eta} &= - \frac{a_o^2}{4\sigma} g^{1/2} (mx)^{-3/2} + \frac{a_o^2 \sigma}{4} (gmx)^{-1/2} \\ &= \bar{\eta}_1 + \frac{a_o^2 \sigma}{4} (gmx)^{-1/2} . \qquad (64) \end{aligned}$$

Hence, the sloping bottom solution results in slightly greater set down as compared with linear theory. Numerical

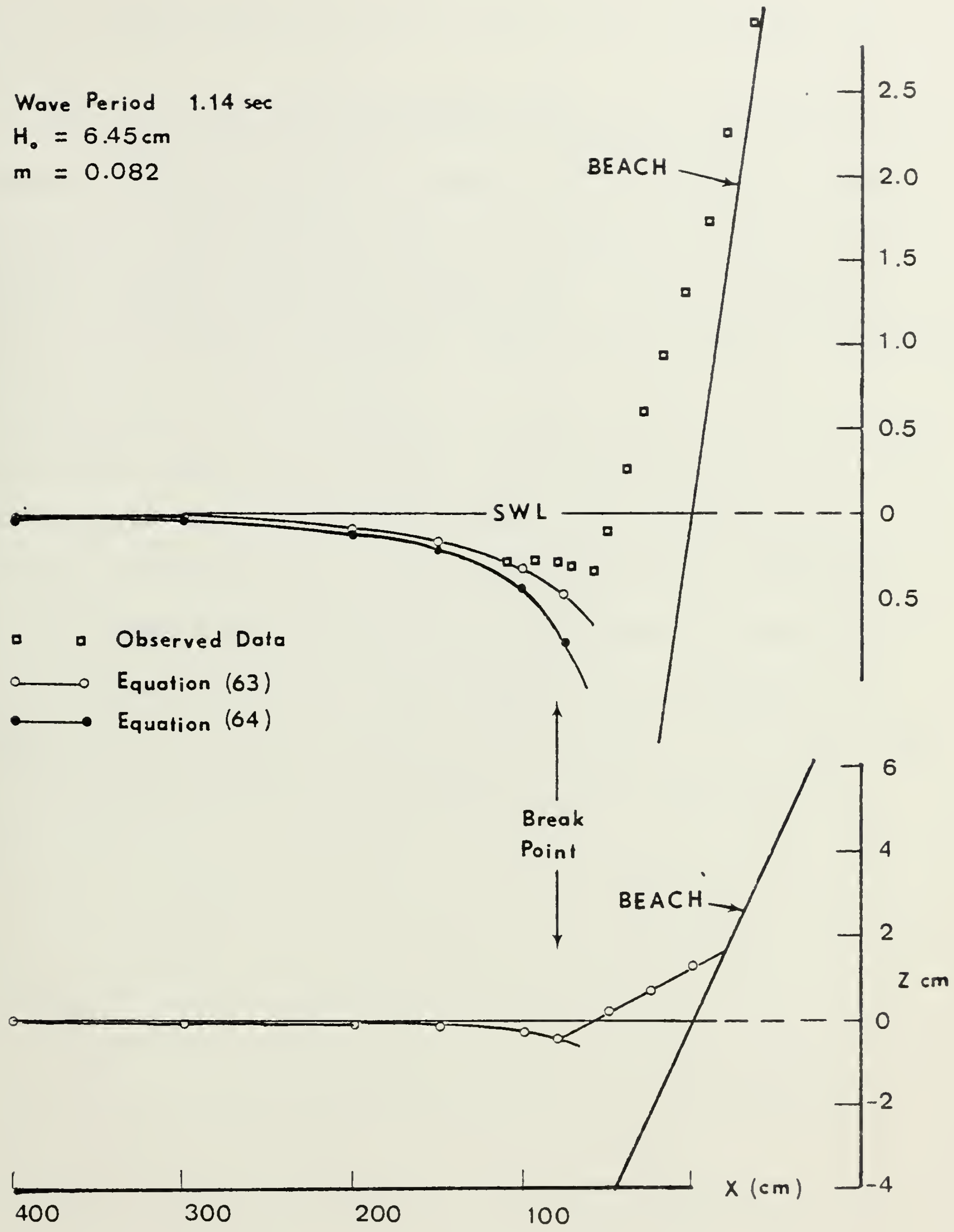

Figure 2: Profile of Mean Sea Level

results of these equations, (63) and (64), are plotted in Figure 2.

Close to the break point, most theoretical results fail to compare favorably with the experimental results. Theory increases set down rapidly near the break point while experimental results tend to flatten out. This is due most likely to the failure of linear theory to adequately describe breaking waves.

Although simplistic in form, the application of the two frequency wave spectrum demonstrated that fluctuating values are obtained for the set down and set up phenomena, i.e., time dependent solutions were obtained for these concepts. Hence, a group of waves, similar in frequency can be expected to produce a periodic variation in the mean sea level which becomes most apparent when they encounter a beach.

V. CONCLUSIONS

In this study, unsteady solutions for set down and set up were derived using a simple two component wave spectrum. The spectral wave components were described using linear wave theory for both horizontal and sloping bottoms. The sloping bottom solution involving the Bessel functions gave a slightly greater set down compared with that of the linear wave theory solution. However, both unsteady solutions for set down showed, that to at least a first approximation, a steady and a fluctuating component, the steady component being identical to the earlier steady state solutions.

The set down at the breaker line acts as the boundary condition driving the set up inside the surf zone. The set down solution showed that the steady and unsteady components are simply additive. Hence, inside the surf zone a solution is composed of the steady set up and a long wave, i.e., surf beat, which is driven by the fluctuating condition at the breaker line.

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